EXAM I, MA3025, Spring 2004

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1. Let the universe of discourse be the set $\{-1, 0, 1\}$. Determine the truth value of each of the following (explain your conclusion):

   (a) $\forall x \exists y(|x| = y)$

   (b) $\exists x \forall y(|x| = y)$

   (c) $\forall x, y \exists z(x + y = z)$

   (d) $\exists x \forall y(xy = 0)$

2. A *perfect square* is defined to be the square of an integer. Prove that, for all integers m and n, if m and n are perfect squares then mn is a perfect square. Does the converse hold?

3. It can be shown that if $k \neq 0$ and $k \neq -1$, then $\dfrac{1}{k(k+1)} = \dfrac{1}{k} - \dfrac{1}{k+1}$. Use this identity to discover a formula for $\displaystyle\sum_{k=1}^{n} \frac{1}{k(k+1)}$.

4. Translate the following argument into symbols, and determine whether it is valid:

   > "If Dominic goes to the racetrack, then Helen will be mad. If Ralph plays cards all night, then Carmela will be mad. If either Helen or Carmela gets mad, then Veronica (their attorney) will be notified. Veronica has not heard from either Helen or Carmela. Therefore, Dominic didn't go the racetrack and Ralph didn't play cards all night."

5. Prove or disprove that, for all sets A and B, $(A - B) - C = (A - C) - (B - C)$.)